would be the Ptolemaic and not the Copernican theory that would permit the comparison of the radii of planetary orbits by Copernicus's geometrical methods. Therefore, to take the possibility of comparing the magnitude of planetary orbits as an indication of the merit of one system over the other is to attribute fundamental significance to the location of humans in the planetary system. Such an attribution may have been appropriate within Aristotelian theory, but it is hardly appropriate now.

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A PROBLEM FOR RELATIVE INFORMATION MINIMIZERS IN PROBABILITY KINEMATICS

1 Williams [1980] provides a persuasive and insightful presentation of the view that the correct general principle for probability kinematics is the one that requires minimising the information content of the posterior probability function relative to the prior. This is a view which I have also presented sympathetically [1980] in a similar context. Further reflection suggests, however, that this Principle needs more intuitive support than it has thus far received. I shall here briefly restate the reasons we have for thinking the principle correct, and then give the sort of example which shows that these reasons do not yet go far enough.

2 Let the agent's prior belief state be characterised by the probability function P and his posterior state by P', defined on the same probability space. Given any measurable partition X on which P is positive (I shall restrict the discussion to this case), the relative information in P' with respect to P as measured in X is defined by

$$I(P', P, X) = \Sigma\{P'(A)\log(P'(A)/P(A)) \colon A \in X\} \qquad (2\text{-}1)$$

The deliverances of experience place a constraint on what the posterior P' should be. For example, in a simple learning experience, the agent accepts the constraint that $P'(E) = 1$, for some proposition E. In the observation-by-candlelight examples discussed by Richard Jeffrey [1965] he accepts the simultaneous constraints that $P'(A) = g_A$ for each element A of some partition X. Further possible constraints are that the posterior conditional probability $P'(A|B)$ be r or that the posterior odds $P'(A)/P'(B)$ be s, or that the posterior expectation $\mathrm{Ex}P'(g)$ be w, where g is some random variable. The last sort is the most general in that all the others can be reformulated in that form.

The *Infomin Principle*, as I shall call it, now says that the agent should choose his posterior P' so as to satisfy that constraint while minimizing information relative to P. As measured in what partition? Let us restrict our discussion further to the case where there is a finite coarsest relevant partition. In the most general case, the random variable g is constant on each element of the partition, but has different values in different elements thereof. If X is that partition, we can first minimise $I(P', P, X)$; *and then* minimise $I(P', P, X')$ for all refinements X' of X by setting $P'(-|A) = P(-|A)$ for each A in X. Thus the problem is reduced to minimising relative information in the coarsest relevant partition.

3 What reasons have we for thinking that the Infomin Principle is correct?

(I) In the simple learning case, it gives the same answer as the Bayesian principle of conditionalisation, $P' = P(-|E)$.

(II) In the observation-by-candlelight case, it leads to the same result as Jeffrey's rule, $P' = \Sigma\{g_A P(-|A) : A \in X\}$.

(III) If the explication of information content is adequate, then the Infomin Principle is the rule that one should not jump to unwarranted conclusions, or add capricious assumptions, when accommodating one's belief state to the deliverances of experience.

While not jumping to conclusions is a very conservative rule, with no room for bold conjectures or acceptance of merely confirmed hypotheses, one could not very well generalise Bayesian procedures by less conservative means. The weak link in the justification lies of course in the antecedent of III. This can be further supported by reasoning that motivates definition (2-1) as by Hobson [1971] and others. This reasoning is rather abstract, and relies on the acceptance of general desiderata for any definition of information content, which do not strike everyone as equally modest.

There is however an obvious course of action for further testing of Infomin against our intuitions. That is to see what posteriors it prescribes when we have simple constraints of sorts not covered by (I) and (II). When the constraint is a posterior conditional probability or posterior odds for two given propositions—cases which appear to be common at race-tracks, in lying spy examples, and so forth—this might be expected to work well. I have found that in these cases, the rather complicated general formulas of Infomin can be simplified so that a pocket calculator is all the agent needs to police his posterior degrees of belief.

4 In the recent movie *Private Benjamin*, Goldie Hawn enters the army and during war games, she and her patrol are dropped in a swampy area which

they have to patrol. I shall now continue the story of their exploits there without straying too far from the movie. The war games area is divided into the region of the Blue Army, to which Judy Benjamin and her fellow soldiers belong, and that of the Red Army. Each of these regions is further divided into Headquarters Company Area and Second Company Area. The patrol has a map which none of them understands, and they are soon hopelessly lost. Using their radio they are at one point able to contact their own headquarters. After describing whatever they remember of their movements, they are told by the duty officer 'I don't know whether or not you have strayed into Red Army territory. But if you have, the probability is 3/4 that you are in their Headquarters Company Area.' At this point the radio gives out.

The first relevant partition is the cross classification Red/Blue; Headquarters/Second. The obvious prior P to assume assigns each the resulting four areas a probability of one quarter. But this is not the coarsest relevant partition for the constraint, which does not pertain to subdivisions of the friendly Blue Army territory. The coarsest description is therefore:

$$A_1 = \text{Red Second Company Area}$$
$$A_2 = \text{Red Headquarters Company Area}$$
$$A_3 = \text{Blue Army Region}$$

$$\text{prior: } P(A_1) = 1/4 = P(A_2)$$
$$P(A_3) = 1/2$$
$$\text{posterior: } P'(A_2|A_1 \text{ or } A_2) = 3/4$$

Question: What will be Private Benjamin's posterior probability that she is in the friendly Blue Army Region?

That is equivalent to the question what her posterior probability function is, and the Infomin Principle will answer it. To see how, let us describe the example more generally.

5 For simplicity I shall discuss the three element partition $X = \{A_1, A_2, A_3\}$, and for generality let the prior be $P(A_i) = p_i$ and the constraint $P'(A_2|A_1 \text{ or } A_2) = q$. This constraint can be stated in three ways:

(1) $P'(A_2) \div (P'(A_1)+P'(A_2)) = q$

(2) $P'(A_2) \div P'(A_1) = q \div (1-q)$

(3) Expectation $P'(\pi) = q$ where π is the random variable which is constant on each element of X with values:

X	π
A_1	$0 = \pi_1$
A_2	$1 = \pi_2$
A_3	$q = \pi_3$

The second formulation formulates it as a constraint on the posterior odds. Let me call an *odds vector* for a probability function p on a finite partition Y

$= \{B_1, \ldots, B_n\}$ any n-tuple of real numbers of form $\langle mp(B_1); \ldots; mp(B_n)\rangle$ where m is any positive real number. When $m = 1$, the vector is singled out by the fact that its elements sum to 1, and I shall call it *normalised*. The odds vector singled out by the fact that its first element equals 1, I shall call *canonical*. The posterior odds vector is constrained by (2) above to have form

$$v' = \langle 1-q; q; y\rangle \tag{5-1}$$

where y is unknown. Let the canonical prior odds vector be

$$v = \langle 1; s; w\rangle. \tag{5-2}$$

The Infomin Principle, given (5-2) and the constraint formulated as in (3) above, should now allow us to determine the unknown.

Fed data (3), the Principle gives the following answer:

$$P'(A_i) = \frac{e^{m\pi_i} P(A_i)}{Z} \tag{5-3}$$

where m is some constant, and Z a factor inserted solely to make P' sum to unity on X. Hence (5-3) tells us that the posterior odds vector must take the form:

$$\begin{aligned} v'' &= \langle (e^{m0} \cdot 1); (e^{m1} \cdot s); (e^{mq} \cdot w)\rangle \\ &= \langle 1; e^m s; e^{mq} w\rangle \end{aligned}$$

But v' and v'' are both positive multiples of a single vector. Hence the proportions between their corresponding elements must be the same.

$$\frac{q}{1-q} = \frac{e^m s}{1}; i.e.\, e^m = q/s(1-q). \tag{5-5}$$

We can now insert numerical values when prior and constraint are known, without needing to calculate exponentials or logarithms.

6 In the Judy Benjamin example we have the normalised odds vector $\langle 1/4, 1/4, 1/2\rangle$ or in a canonical form, $\langle 1, 1, 2\rangle$. Thus $s = 1$ and $w = 2$. Also $q = 3/4$, so $q/1-q = 3$ and thus $e^m = 3s = 3$ as well. Substituting these values in (5-4) we arrive at

$$v'' = \langle 1, 3, (3)^{3/4} \cdot 2\rangle \tag{6-1}$$

Because the fourth root of 3 is approximately 1.316, that last number equals approximately 4.558. The normalised odds vector then gives us the approximate values:

$$\begin{aligned} P'(A_1) &= 0.117 \\ P'(A_2) &= 0.351 \\ P'(A_3) &= 0.532 \end{aligned}$$

The glaring feature of this solution is that the adjustment of Judy Benjamin's conditional probabilities in the Red Army Region have increased her subjective probability that she is still in friendly Blue Army Territory!

This effect is of course aggravated if we make the value of q more extreme, say 7/8. Then $e^m = 7$ and we get

$$v'' = \langle 1, 7, (7)^{7/8} \cdot 2 \rangle \tag{6-2}$$

The eighth root of 7 is approximately 1.275, so the last number is approximately 11 and the probability of A_3 (being in the friendly Blue Region) becomes approximately equal to 0.58. More generally, we notice that by starting with an original conditional probability of 1/2, the posterior odds vector must always take the form

$$v'' = \langle 1, n, (n)^{n/n+1} \cdot 2 \rangle \tag{6-3}$$

if we let $q = n/n+1$. This clearly approaches $\langle 1, n, 2n \rangle$ as we increase n, with the posterior probability for being in the Blue Region thus tending to $2n/3n + 1$, which in turn approaches 2/3.

It is hard not to speculate that the dangerous implications of being in the enemy's headquarters area, are causing Judy Benjamin to indulge in wishful thinking, her indulgence becoming stronger as her conditional estimate of the danger increases.[1]

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[1] [*Added in press*]: I wish to thank Dr Williams for a number of valuable comments he made on this discussion note, in correspondence. While it was too late for the printed version to benefit from these, I should like to mention one point especially. It refers to my remark near the end, that the posterior probability for being in the Blue Region tends to the limit 2/3 as input number q approaches 1. Dr Williams pointed out that the limiting case is exactly the one in which Private Benjamin becomes sure that she is not in the Red Second Company Area. This result agrees then with Bayesian Conditionalisation on that proposition, yielding the posterior $P'(-) = P(-|\text{not } A_1)$. It seems now to me that we may very well have the beginnings here of the required further justification for *Infomin*. For obviously the two procedures should agree in any case where both are applicable, and just as obviously they could not agree in this limiting case if *Infomin* never affected the probability of being in the Blue Region in response to the sort of input in question. Continuity considerations (such as that this posterior probability should vary continuously with q) if themselves sufficiently well motivated, may then help to remove our initial misgivings about the proffered posterior.