

Philosophy 122 Assignment #1
Due by the start of class on Thursday, October 25th

You must do problems 1 and 2. Then complete any 5 out of problems 3-9.

1) Let P be a probability measure on S with $E, F, G \subset S$. Assume that it satisfies:

$$P[E] = 0.3$$

$$P[F] = 0.45$$

$$P[G] = 0.5$$

$$P[E \cup F] = 0.6$$

$$P[E \cup G] = 0.7$$

$$P[F \cup G] = 0.8$$

$$P[E \cap F \cap G] = 0.05$$

Find $P[E \cup F \cup G]$. Show your work in a way that makes it clear how you could have done a slightly different problem. For example, if you use a Venn diagram, tell me the order you filled in the regions and which regions correspond to the answer. If you use the algebraic method, just write the relevant equations, etc.).

2) Define the false positive rate of a test to be the probability of getting a positive result given that the patient does not have the disease. The false negative rate is the probability of getting a negative result given that they do have it.

You overhear a doctor tell her patient: “Now after the last set of tests, I told you I was 75% sure that you had the antibody in your blood so we decided to do another test. Well, now I can say that I am 95% sure. After all, the test came out positive and the false positive rate on this test is only 10%.” Now you know that this doctor is a competent statistician. What can you infer about the false negative rate of the test? If the test had instead come out negative, how should the doctor have revised her beliefs?

3) Bruno de Finetti believed that the proper axioms for probability theory included only finite additivity and not countable additivity. Explain the difference between the two and carefully describe the ‘infinite lottery’ example which he took to be an argument to choose between the two. Is this a good argument?

4) Describe the Principle of Indifference and describe a particular case which is meant to be a problem for it (for example, the cube factory from van Fraassen). Is this a decisive objection to the principle?

5) In Richard Feynman's Lectures on Physics, Volume 1, we find the following “definition” of probability:

By the “probability” of a particular outcome of an observation we mean our estimate for the most likely fraction of a number of repeated observations that will yield that particular outcome.

There are many problems with this definition. Briefly indicate several of them.

6) The Laws of Large Numbers are mathematical theorems that relate the probability of a single event on a single trial to the probability of getting different relative frequencies of given outcomes of repeated trials of the same type of event. But on some interpretations of probability such as von Mises’s version of Frequentism, the probability of an outcome on a single trial doesn’t even make sense. On other kinds of Frequentist views (and on some propensity views), the single trial probability is just defined to be something about this long-run frequency. Can any sense be made of the probability of the single case apart from its long-run behavior? Could these come apart? If so, is this an argument against these interpretations?

7) Do 7a through 7e.

7a) Imagine that I post the following betting odds: 4:1 for the proposition that P. 3:1 for the proposition that Q. And 3:1 against P&Q. Describe a set of three bets all at stakes of one dollar and all of which I consider to be fair [explain why I consider them fair] and prove that if I take all three bets I am guaranteed to lose money no matter what.

7b) Assume P implies $\sim Q$ and that my betting odds for P are a:b and my odds for Q are c:d. What do my betting odds for $P \vee Q$ have to be to guarantee that I am not subject to a Dutch Book? (You may assume that if my betting ratios are probabilities I am not subject to a Dutch Book.)

7c) Assume that my betting odds for $P \vee Q$ are not as given in the previous problem. Show that for any finite value \$M there is a set of three bets that I

consider fair such that if I take all three bets I am guaranteed to lose at least \$M no matter what.

7d) Repeat question 7c: except assume that I do not accept bets that I consider to be fair, only that I consider to be to my advantage (subjective expected value is greater than \$0). Show that I am still subject to a Dutch Book where I will lose at least \$M no matter what.

7e) Assume that I will not accept any collection of bets with stakes that sum to more than \$100. Further assume that I will not accept any bets unless I deem the subjective expected utility of the bet to be at least \$1. Show that it is possible for me to have degrees of belief that violate the axioms where it will not be possible to make a Dutch Book against me.

8) Subjectivists about probability describe various ways of getting at an agent's personal probabilities by asking about preferences between various gambles. The Dutch book argument apparently assumes that for any particular proposition X, an agent will have some degree of belief in X which can be appropriately determined by determining which bets that agent considers fair which is then interpreted as 'would be willing to take either side of the relevant bet'. The argument then proceeds to claim that these degrees of belief must be probabilities. Imagine that for some particular agent and for some X, there is no bet that the agent would be willing to take either side of. Does this mean that there is no bet that the agent considers fair? Does it mean that the agent has no degree of belief with respect to X? Does this represent any kind of failure of the Dutch Book argument? Or something else?

9) The Diachronic Dutch Book argument claims to show that if you have a specific plan for updating your degrees of belief based on new evidence and it is something other than updating by conditionalization, then you could be subject to a diachronic dutch book and a sure loss. Does this necessarily represent a failure of irrationality? At the end of "Why Conditionalize", Lewis points out that if you have no particular plan for how to change your beliefs, this is just a different kind of irrationality. Is this right? Note also that if you do have a particular plan but then you don't actually follow through on it, then you can also avoid a dutch book. Is this also irrational?